

Topologically Massive Non-Abelian Gauge Theories: Constraints and Deformations

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Abstract

We study the relationship between three non-Abelian topologically massive gauge theories, viz. the naïve non-Abelian generalization of the Abelian model, Freedman-Townsend model and the dynamical 2-form theory, in the canonical framework. Hamiltonian formulation of the naïve non-Abelian theory is presented first. The other two non-Abelian models are obtained by deforming the constraints of this model. We study the role of the auxiliary vector field in the dynamical 2-form theory in the canonical framework and show that the dynamical 2-form theory cannot be considered as the embedded version of naïve non-Abelian model. The reducibility aspect and gauge algebra of the latter models are also discussed.

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I. INTRODUCTION

Construction and study of gauge invariant theories of massive vector fields has been a problem of great intrinsic interest. Such theories also have a potential application because the Higgs particle, needed for giving masses to gauge fields and fermions in the standard model, does not yet have experimental support. Consequently, alternative theories which have no residual Higgs scalar, for both Abelian and non-Abelian gauge fields, deserve closer attention. One of the oldest models in which there is no residual scalar particle is the Stückelberg formulation. Another gauge invariant model where massive gauge fields appear couples a vector field to a second rank anti-symmetric tensor field through a topological $B \wedge F$ term. The Abelian theory is described by the Lagrangian [1–3]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}F^{\mu\nu}B^{\lambda\sigma}, \quad (1.1)$$

where $H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \text{cyclic terms}$. This Lagrangian is invariant under two independent gauge transformations,

$$\begin{aligned} \delta A_\mu &= \partial_\mu \omega, \\ \delta B_{\mu\nu} &= (\partial_\mu \lambda_\nu - \partial_\nu \lambda_\mu). \end{aligned} \quad (1.2)$$

The equations of motion following from the above Lagrangian are

$$\begin{aligned} \partial^\nu F_{\mu\nu} - \frac{m}{6}\epsilon_{\mu\nu\lambda\sigma}H^{\nu\lambda\sigma} &= 0, \\ \partial^\lambda H_{\mu\nu\lambda} - \frac{m}{2}\epsilon_{\mu\nu\lambda\sigma}F^{\lambda\sigma} &= 0, \end{aligned} \quad (1.3)$$

which are like the London equations of superconductivity, and has the interpretation of a massive vector. The massive nature of the vector boson can also be brought out by summing over propagators, which leads to the appearance of a pole in the vector propagator [3].

Making a non-Abelian model of massive vector bosons using this mechanism is a non-trivial task. Naïvely, one can replace the ordinary derivative ∂_μ with the gauge-covariant derivative D_μ in the Lagrangian of Eqn. (1.1) to get

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{12}H_{\mu\nu\lambda}^a H^{a\mu\nu\lambda} + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma} B^{a\mu\nu} F^{a\lambda\sigma}, \quad (1.4)$$

where B lives in the adjoint representation of the gauge group and $H_{\mu\nu\lambda}^a = (D_\mu B_{\nu\lambda})^a + \text{cyclic terms}$. But although this non-Abelian model is invariant under the usual gauge transformations

$$\delta A_\mu^a = (D_\mu \omega)^a, \quad \delta B_{\mu\nu}^a = g f^{abc} B_{\mu\nu}^b \omega^c, \quad (1.5)$$

unlike the Abelian model it is not invariant under the non-Abelian vector gauge transformations of the 2-form,

$$\delta B_{\mu\nu}^a = (D_\mu \lambda_\nu - D_\nu \lambda_\mu)^a. \quad (1.6)$$

The absence of the vector gauge symmetry makes perturbative calculations from the Lagrangian of Eqn. (1.4) quite problematic. Gauge-fixing for $B_{\mu\nu}^a$ is not needed in the absence of the symmetry, but the quadratic part of the kinetic term for $B_{\mu\nu}^a$ cannot be inverted without a gauge-fixing term.

The problem runs even deeper, and has been the topic of a recent ‘no-go’ theorem [4] which uses consistent deformation of the master equation in the antifield formalism. This theorem states that there is no perturbatively renormalizable non-Abelian generalization of Eqn. (1.1) with the same field content. This is a strong result, but there are two non-Abelian topologically massive models which evade the strictures of this theorem because their field content are different from that of Eqn. (1.4) even though their Abelian limits are either Eqn. (1.1) or an equivalent first order formulation. The modified field contents ensure that both these models are invariant under the vector gauge transformations of Eqn. (1.6).

The first of these is the the Freedman-Townsend model described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{m}{2}\Phi_\mu^a \Phi^{a\mu} + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma} F(v)^{a\mu\nu} B^{a\lambda\sigma}, \quad (1.7)$$

where $v_\mu = A_\mu + \Phi_\mu$, $F_{\mu\nu}^a$ is the usual Yang-Mills field strength of A_μ and $F_{\mu\nu}^a(v)$ is the Yang-Mills field strength calculated for v_μ . The non-Abelian 2-form B acts as an auxiliary

field in this model, forcing v to be a flat connection. Quantization of Freedman-Townsend model has been studied using B-V formalism [5] and shown that it is unitary but plagued with non-renormalizable propagators. It has been shown recently that by using the self-interaction mechanism, a first-order form of Eqn. (1.1) gives rise to Freedman-Townsend Lagrangian [6].

The second one is the theory of the dynamical 2-form [7] given by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{12}H_{\mu\nu\lambda}^a H^{a\mu\nu\lambda} + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma} B^{a\mu\nu} F^{a\lambda\sigma}, \quad (1.8)$$

where $H_{\mu\nu\lambda}^a$ is now the compensated field strength, invariant under non-Abelian vector gauge transformations, $H_{\mu\nu\lambda}^a = \partial_\mu B_{\nu\lambda}^a + g f^{abc} A_\mu^b B_{\nu\lambda}^c + g f^{abc} C_\mu^b F_{\nu\lambda}^c + \text{cyclic terms}$. The quantization of this model has been studied in the BRST/anti-BRST scheme [8,9]. A proof of renormalizability of this model was recently constructed [10] as well. Both these models have more fields than the naïve non-Abelian model of Eqn. (1.4), and as a result can circumvent the no-go theorem of [4] and remain invariant under the non-Abelian vector gauge transformation, given in Eqn. (1.6).

The purpose of this paper is threefold — (i) to present the Hamiltonian analysis of the naïve non-Abelian model (1.4), (ii) to see if, by any procedure one can modify the second class constraints of the naïve non-Abelian model to first class such that the modified theory will be invariant under the vector gauge transformations (1.6), and (iii) to investigate the role of the auxiliary field in (1.8) through the analysis of constraints. The Hamiltonian formulation of non-Abelian topologically massive gauge theories is interesting by itself and to the best of our knowledge has not been studied in detail. With these motivations, we have done the Hamiltonian analysis of the model of Eqn. (1.4). Then we address the question whether we can elevate, by any procedure, the naïve model to a theory symmetric under the vector gauge transformations of Eqn. (1.6).

Generally a theory without any gauge symmetry can be converted to one with a gauge symmetry by using the generalized canonical scheme developed by Batalin, Fradkin, Tyutin (BFT) [11,12] and collaborators. In this method, the phase space is first enlarged by in-

Introducing a pair of canonically conjugate variables for each second class constraint. Using these new variables, the constraints are modified so that they have vanishing Poisson brackets among themselves. Then the Hamiltonian is modified to have vanishing Poisson brackets with all the modified constraints. This procedure is systematically iterated until all second class constraints are converted to first class and a gauge invariant Hamiltonian and a nilpotent BRST charge are constructed. The new fields introduced in BFT scheme are usually identified with the Stückelberg fields and their momenta. In this method one recovers the original system with second class constraints by setting the newly introduced variables to zero. In the path integral approach, by starting with the phase space partition function of the embedded model one can obtain that of the original model showing their equivalence. In order to do this, one has to choose either the newly introduced BFT fields or equivalently the second class constraints of the original model as gauge fixing conditions (unitary gauge) [11,12] corresponding to the first class constraints of the embedded model. But the application of this method to non-Abelian theories is more involved and there is an additional complication for the theory of Eqn. (1.4). As we shall argue later, BFT embedding of the model in Eqn. (1.4) may lead to a non-local theory.

In this paper we start from the Hamiltonian and the constraints of Eqn. (1.4). We wish to see if the vector gauge symmetry which was present in the Abelian model can be restored in the form of Eqn. (1.6) in any non-Abelian generalization. This is done by deforming a second class constraint, keeping the canonical Hamiltonian unchanged, so that the modified constraint is first class and it generates the vector symmetry transformation. This may require the modification of the other constraints also, but the deformation is done in such a way that the existing $SU(N)$ gauge symmetry is not lost. Finally, given the constraints and the Hamiltonian, the Lagrangian from which they follow has to be calculated. This job is made easy for the model at hand, as we already know of two non-Abelian models, namely Freedman-Townsend model and dynamical 2-form theory, where the vector symmetry is present.

In this procedure, the original constraints are deformed by the use of the original phase

space variables as well as new ones, which is similar to the BFT procedure, but we look for a local theory at the end. Although this method as discussed seems to be applicable only to this system at this time, it provides a clearer understanding of the contrasting features and nature of constraints of these two models of vector mass generation. The spirit of this method may also be useful in dealing with other theories which lead to non-local field theories via the BFT procedure.

The dynamical 2-form theory of Eqn. (1.8) has an auxiliary vector field which undergoes a transformation, compensating for the vector gauge transformation of the $B_{\mu\nu}^a$ field as given in Eqn. (1.6). In this sense, the auxiliary field resembles a Stückelberg field. Typically Stückelberg fields are introduced to compensate the non-invariance of a mass term. But here it is the kinetic term of the $B_{\mu\nu}^a$ field, rather than the (topological) mass term, which is not invariant under Eqn. (1.6) and requires the auxiliary field in order to restore invariance. Another difference between a Stückelberg field and the auxiliary field is that a Stückelberg field generally has a kinetic term while the auxiliary field here does not have any. Thus in the role of a compensating field, the auxiliary field appears just as in the Stückelberg mechanism, but its properties are very different from those of a Stückelberg field. In the course of our Hamiltonian analysis of the system, we shall investigate the role of the auxiliary field in the model of Eqn. (1.8) in the Hamiltonian formulation.

This paper is organized as follows. In Sec. II, we present the canonical analysis of a first-order formulation of the theory of Eqn. (1.4) and show that it has first class as well as second class constraints. Using the constraints and their Poisson brackets we then argue that BFT embedding can lead to non-local theory. In Sec. III), we deform the constraints using only the original phase space variables so that the vector gauge symmetry is implemented by first class constraints. There we show that the deformed system is equivalent to the Freedman-Townsend model. Next in Sec. IV, we deform the constraints using the original as well as newly introduced canonically conjugate variables, whereby we obtain the dynamical 2-form theory. In Sec V, we discuss the role played by the auxiliary C_μ field in dynamical 2-form theory of Eqn. (1.8). We also show here that the model in Eqn. (1.8) cannot be the

embedded version of the naïve model of Eqn. (1.4). We conclude with a comparative study of the Freedman-Townsend model and dynamical 2-form theory and discussions in Sec. VI.

Conventions: We use the metric $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ and $\epsilon_{0123} = 1$. We shall take the gauge group to be $SU(N)$, with generators t^a satisfying

$$\begin{aligned} [t^a, t^b] &= if^{abc}t^c, \\ \text{tr}(t^a t^b) &= \frac{1}{2}\delta^{ab}. \end{aligned} \quad (1.9)$$

The field strength F of a gauge field A is defined as

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c. \quad (1.10)$$

II. NAÏVE NON-ABELIAN MODEL: HAMILTONIAN ANALYSIS

In this section we consider Eqn. (1.4) in a first-order formulation. We analyze the Hamiltonian structure of the system. The non-invariance of the model under the vector gauge transformation of Eqn. (1.6) is reflected by the second class nature of the constraint which implements this symmetry. By deforming this constraint using only the original phase space variables we can convert this constraint to a first class one. This requires modification of the remaining constraints as well so as to leave the first class or second class nature of all the other constraints unchanged. Thus this deformation gives us a new gauge system which is invariant under the $SU(N)$ gauge symmetry of Eqn. (1.5), as well as the vector gauge symmetry of Eqn. (1.6). We shall find that the new system is identical to the Freedman-Townsend model.

We start the Hamiltonian analysis from the first-order Lagrangian

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{m^2}{2}\Phi_\mu^a \Phi^{a\mu} + \frac{m}{2}\epsilon_{\mu\nu\lambda\sigma}\Phi^{a\mu}(D^\nu B^{\lambda\sigma})^a \\ & + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}A^{a\mu}(D^\nu B^{\lambda\sigma})^a + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}A^{a\mu}\partial^\nu B^{a\lambda\sigma}. \end{aligned} \quad (2.1)$$

By integrating out the Φ_μ^a field from the above Lagrangian we get the second-order Lagrangian of Eqn. (1.4). We have not put the gauge potential part in first order form. The primary constraints following from this Lagrangian are

$$\Pi_0^a \approx 0, \quad \tilde{\Pi}_0^a \approx 0, \quad \Pi_{0i}^a \approx 0, \quad \tilde{\Pi}_i^a \approx 0,$$

$$\Lambda_{ij}^a = \Pi_{ij}^a + m\epsilon_{ijk}(A^{ak} + \Phi^{ak}) \approx 0, \quad (2.2)$$

where $\epsilon_{ijk} = \epsilon_{0ijk}$, and we have written $\Pi_\mu^a, \tilde{\Pi}_\mu^a, \Pi_{\mu\nu}^a$ for the momenta canonically conjugate to $A^{a\mu}, \Phi^{a\mu}$ and $B^{a\mu\nu}$, respectively. The total Hamiltonian is then

$$\mathcal{H}_T = \frac{1}{4}F_{ij}^a F^{aij} - \frac{1}{2}\Pi_i^a \Pi^{ai} - \frac{m^2}{2}\Phi_i^a \Phi^{ai} - A_0^a \tilde{\Lambda}^a - \Phi_0^a \tilde{\omega}^a - B^{a0i} \tilde{\Lambda}_i^a. \quad (2.3)$$

The coefficients of the Lagrange multipliers $A^{a0}, \Phi^{a0}, B^{a0i}$ are the secondary constraints which appear upon demanding the persistence of the primary constraints,

$$\tilde{\Lambda}^a = \{\Pi_0^a, \mathcal{H}\} = (D^i \Pi_i)^a + \frac{m}{2}\epsilon_{ijk}(D^i B^{jk})^a + \frac{m}{2}gf^{abc}\epsilon_{ijk}\Phi^{bi}B^{cjk}, \quad (2.4)$$

$$\tilde{\Lambda}_i^a = \{\Pi_{0i}^a, \mathcal{H}\} = \frac{m}{2}\epsilon_{ijk}(\Phi^{ajk} + F^{ajk}), \quad (2.5)$$

$$\tilde{\omega}^a = \{\tilde{\Pi}_0^a, \mathcal{H}\} = m^2\Phi^{a0} + \frac{m}{2}\epsilon_{ijk}(D^i B^{jk})^a. \quad (2.6)$$

All these secondary constraints are second class as can be checked easily from their Poisson brackets with one another. But we can construct their linear combinations which are first class at this stage. Let us define one combination which is going to be the Gauss law constraint, and another which will generate the vector gauge transformations,

$$\Lambda^a = \tilde{\Lambda}^a - \frac{1}{2}gf^{abc}\Lambda_{ij}^b B^{cij} - gf^{abc}\tilde{\Pi}_0^b \Phi^{c0} - gf^{abc}\tilde{\Pi}_i^b \Phi^{ci}, \quad (2.7)$$

$$\bar{\Lambda}_i^a = \tilde{\Lambda}_i^a - (D^j \Lambda_{ij})^a. \quad (2.8)$$

At this stage, the Poisson brackets among the constraints are

$$\begin{aligned} \{\Lambda^a, \Lambda^b\} &= -gf^{abc}\Lambda^c, \quad \{\Lambda^a, \bar{\Lambda}_i^b\} = gf^{abc}\Lambda_i^c, \quad \{\Lambda^a, \tilde{\omega}^b\} = gf^{abc}\tilde{\omega}^c, \\ \{\Lambda^a, \Lambda_{ij}^b\} &= gf^{abc}\Lambda_{ij}^c, \quad \{\Lambda^a, \tilde{\Pi}_i^b\} = gf^{abc}\tilde{\Pi}_i^c, \quad \{\Lambda^a, \Pi_0^b\} = 0, \quad \{\Lambda^a, \Pi_{0i}^b\} = 0, \end{aligned} \quad (2.9)$$

$$\begin{aligned} \{\bar{\Lambda}_i^a, \bar{\Lambda}_j^b\} &= 0, \quad \{\bar{\Lambda}_i^a, \tilde{\omega}^b\} = 0, \quad \{\bar{\Lambda}_i^a, \Pi_0^b\} = 0, \quad \{\bar{\Lambda}_i^a, \tilde{\Pi}_0^b\} = 0, \quad \{\bar{\Lambda}_i^a, \tilde{\Pi}_j^b\} = 0, \\ \{\bar{\Lambda}_i^a, \Lambda_{ij}^b\} &= 0, \quad \{\tilde{\Pi}_0^a, \tilde{\omega}^b\} = -m^2, \\ \{\tilde{\Pi}_i^a, \Lambda_{ij}^b\} &= -m\delta^{ab}\epsilon_{ijk} \quad \{\tilde{\omega}^a, \Lambda_{ij}^b\} = m\epsilon_{ijk}D^{abk}\delta(x-y). \end{aligned} \quad (2.10)$$

Note that both Λ^a and $\bar{\Lambda}_i^a$ have vanishing Poisson brackets with all the other constraints as well as with themselves. The constraint Λ^a is preserved, $\{\Lambda^a, \mathcal{H}\} = 0$, while

$$\{\bar{\Lambda}_i^a, \mathcal{H}\} = g f^{abc} \Pi^{bj} \Lambda_{ij}^c - m g f^{abc} \epsilon_{ijk} \Pi^{bj} \Phi^{ck} \equiv \psi_i^a \quad (2.11)$$

is a tertiary constraint, which has non vanishing Poisson bracket with $\bar{\Lambda}_i^a$ and $\tilde{\omega}^a$. Now the first class constraints are Π_0^a, Π_{0i}^a and Λ^a , and the second class constraints are $\tilde{\omega}_i^a, \Lambda_{ij}^a, \omega^a, \tilde{\Pi}_0^a, \bar{\Lambda}_i^a, \psi_i^a$ which together remove 24 out of 28 phase space degrees. Thus a naïve counting shows that the above model has only four phase space degrees of freedom and cannot describe a massive spin-one theory. On the other hand, the free part of the action (i.e., the $g \rightarrow 0$ limit) coincides with the Abelian action, which has six phase space degrees of freedom and in fact describes a massive vector field. It is therefore possible that there is a reducibility among second class constraints which is not manifest, in which case the model may still describe massive spin-one particles.

The second class nature of the constraint $\bar{\Lambda}_i^a$ is expected since the model of Eqn. (1.4), and therefore Eqn. (2.1), does not have the vector gauge invariance of Eqn. (1.6). Generally, by applying the BFT procedure, a theory with second class constraints T_α (with matrix of Poisson brackets $\{T_\alpha, T_\beta\} = \Delta_{\alpha\beta}$) and Hamiltonian can be converted to a theory with only first class constraints and gauge invariant Hamiltonian. In this method one first enlarges the phase space by introducing auxiliary variables θ_α corresponding to each of the second class constraints. These variables satisfy

$$\{\theta_\alpha, \theta_\beta\} = \omega_{\alpha\beta}, \quad (2.12)$$

which may be taken to be field-independent, and $\omega_{\alpha\beta}$ is such that $\det|\omega_{\alpha\beta}| \neq 0$. Now we define the first class constraints $\bar{T}_\alpha(P, Q, \theta_\alpha)$ (where P and Q stand for the original canonically conjugate phase space variables) in the extended phase space, satisfying

$$\{\bar{T}_\alpha, \bar{T}_\beta\} = 0. \quad (2.13)$$

The solution for this is obtained in a series form as

$$\bar{T}_\alpha = T_\alpha + X_{\alpha\beta}\theta^\beta + \text{higher order terms in } \theta_\alpha, \quad (2.14)$$

where $X_{\alpha\beta}$ satisfy

$$X_{\alpha\beta}\omega^{\beta\lambda}X_{\lambda\rho} = \Delta_{\alpha\rho}. \quad (2.15)$$

After converting the second class constraint to strongly involutive ones, one proceeds to construct the gauge invariant Hamiltonian $\bar{H}(P, Q, \theta_\alpha)$ in the extended phase space. This gauge invariant Hamiltonian has to satisfy

$$\{\bar{T}_\alpha, \bar{H}\} = 0. \quad (2.16)$$

Solving the above equation gives \bar{H} in a series form.

The first class constraints and gauge invariant Hamiltonian are calculated by solving Eqs. (2.14) and (2.16) in BFT procedure by iteration. However, to solve Eqs. (2.14) and (2.16) one needs the inverse of both $\omega_{\alpha\beta}$ and $X_{\alpha\beta}$. In the case of the naïve model, it can be seen directly from Eqn. (2.10) that the matrix $\Delta_{\alpha\beta}$ of the Poisson brackets between the second class constraints (T_α) involve derivatives of delta functions. Because of this $X_{\alpha\beta}$ will also have derivatives of delta functions (see Eqn. (2.15)) and therefore its inverse is likely to be non-local. This will result in the non-locality of the first class constraints \bar{T}_α and gauge invariant Hamiltonian \bar{H} . Thus from the constraint structure of the naïve non-Abelian model, we see that the usual BFT embedding will very likely lead to a non-local theory.

For the sake of completeness we should mention that there is an alternate procedure of converting theories with second class constraints to theories with only first class constraints known as the gauge unfixing method [13]. In all known examples, this procedure and BFT embedding result in the same first class theory. In Appendix A, we apply the gauge unfixing procedure to naïve non-Abelian model of Eqn. (1.4), and find that it too fails to give a first-class theory.

III. FREEDMAN-TOWNSEND MODEL

Since we want the modified theory to be local as well as invariant under the vector gauge transformation as well as under all the original symmetry transformations of the model, in this section we adopt a different approach to convert the second class constraints. Here we would like to modify the constraints $\bar{\Lambda}_i^a \rightarrow \Lambda_i^a$ such that $\{\Lambda_i^a, \mathcal{H}\} \approx 0$ as well as $\{\Lambda_i^a, \chi\} \approx 0$, for all constraints χ in the theory. Since it is only ψ_i^a which has a non-vanishing Poisson bracket with $\bar{\Lambda}_i^a$, we start by modifying $\bar{\Lambda}_i^a$ such that $\{\bar{\Lambda}_i^a, \psi_j^b\} \approx 0$. But this modification will change the other Poisson bracket relations in Eqs. (2.9) and (2.10). In order to keep those initial first class constraints as first class and also to have Λ_i^a first class, we have to further modify Λ_i^a as well as other constraints. For the sake of convenience, let us define $v_i^a = A_i^a + \Phi_i^a$ and $D(v)^{abi} = \delta^{ab}\partial^i + g f^{acb} v^{ci}$. Then the modified constraints read

$$\Lambda^a = (D(A)^i \Pi_i)^a + \frac{m}{2} \epsilon_{ijk} (D(v)^a B^{jk})^a - \frac{g}{2} f^{abc} \Lambda_{ij}^b B^{cij} - g f^{abc} \tilde{\Pi}_0^b \Phi^{c0} - g f^{abc} \tilde{\Pi}_i^b \Phi^{ci}, \quad (3.1)$$

$$\Lambda_i^a = \bar{\Lambda}_i^a + \frac{m}{2} \epsilon_{ijk} f^{abc} \Phi^{bj} \Phi^{ck} - g f^{abc} \Phi^{bj} \Lambda_{ij}^c \quad (3.2)$$

$$\begin{aligned} \omega^a &= \tilde{\omega}^a + \frac{m}{2} g f^{abc} \epsilon_{ijk} \Phi^{bi} B^{cjk} \\ &= m^2 \Phi_0^a + \frac{m}{2} \epsilon_{ijk} (D(v)^i B^{jk})^a. \end{aligned} \quad (3.3)$$

The remaining constraints of Eqn. (2.2) are unchanged. The algebra of the modified constraints is

$$\begin{aligned} \{\Lambda^a, \Lambda^b\} &= -g f^{abc} \Lambda^c, \quad \{\Lambda^a, \Lambda_i^b\} = g f^{abc} \Lambda_i^c, \\ \{\Pi_0^a, \omega^b\} &= -m^2, \quad \{\Lambda_i^a, \Lambda_j^b\} = 0, \\ \{\Lambda^a, \omega^b\} &= g f^{abc} \omega^c, \\ \{\Lambda_i^a, \omega^b\} &= \frac{g}{2} f^{abc} \epsilon_{ijk} F(v)^{cjk}. \end{aligned} \quad (3.4)$$

Note that the last Poisson bracket in the above appears to be still non-vanishing (even weakly), but is not so. This can be seen by noting that Λ_i^a can be expressed as

$$\Lambda_i^a = \frac{m}{2} \epsilon_{ijk} F(v)^{ajk} - (D(v)^j \Lambda_{ij})^a, \quad (3.5)$$

We want the modified theory to be of first-order in $B_{\mu\nu}^a$, and consequently $\Lambda_{ij}^a \approx 0$ should remain a constraint. Therefore the first term in Λ_i^a viz., $\frac{m}{2}\epsilon_{ijk}F(v)^{ajk}$ must be constrained to vanish by itself. Hence the last Poisson bracket in Eqn. (3.4) vanishes weakly. Thus we see that the constraint Λ_i^a is first class. From (3.4) it is clear that Λ^a is first class and ω^a is second class.

Also we notice that

$$(D(v)^i \Lambda_i)^a = -\frac{g}{2} f^{abc} F(v)^{bij} \Lambda_{ij}^c, \quad (3.6)$$

which is zero upon using the constraint $\frac{m}{2}\epsilon_{ijk}F(v)^{ajk} \approx 0$. Thus we see that Λ_i^a is reducible on the constraint surface. Thus among the first class constraints Π_0^a , Π_{0i}^a , Λ^a , Λ_i^a only seven are linearly independent, and we also have eight second class constraints $\tilde{\Pi}_0^a$, $\tilde{\Pi}_i^a$, Λ_{ij}^a , ω^a . Because of these constraints, six phase space degrees of freedom remain, and the system defined by these constraints (2.2, 3.1, 3.2, 3.3) and the canonical Hamiltonian

$$\mathcal{H}_c = \frac{1}{4} F_{ij}^a F^{aij} - \frac{1}{2} \Pi_i^a \Pi^{ai} - \frac{m^2}{2} \Phi_i^a \Phi^{ai} \quad (3.7)$$

describes a massive spin-one model.

The covariant Lagrangian from which this set of constraints and Hamiltonian follow is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{m^2}{2} \Phi_\mu^a \Phi^{a\mu} + \frac{m}{4} \epsilon_{\mu\nu\lambda\sigma} v^{a\mu} (D(v)^\nu B^{\lambda\sigma})^a + \frac{m}{4} \epsilon_{\mu\nu\lambda\sigma} v^{a\mu} \partial^\nu B^{a\lambda\sigma}. \quad (3.8)$$

This Lagrangian can be rewritten after an integration by parts as

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{m^2}{2} \Phi_\mu^a \Phi^{a\mu} + \frac{m}{4} \epsilon_{\mu\nu\lambda\sigma} F(v)^{a\mu\nu} B^{a\lambda\sigma}, \quad (3.9)$$

which is the Freedman-Townsend Lagrangian describing a massive spin-one theory.

IV. DYNAMICAL 2-FORM THEORY

In this section we introduce new pairs of canonically conjugate variables and deform the constraints using these as well as the original phase space variables. The goal of this

deformation is again to turn the constraint of Eqn. (2.5) into a first class constraint. Here again we start our Hamiltonian analysis from a first order Lagrangian

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{m^2}{2}\Phi_\mu^a \Phi^{a\mu} + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}\Phi^{a\mu\nu} B^{a\lambda\sigma} \\ & + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}A^{a\mu}(D^\nu B^{\lambda\sigma})^a + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}A^{a\mu}\partial^\nu B^{a\lambda\sigma},\end{aligned}\quad (4.1)$$

where $\Phi_{\mu\nu}^a = (D_\mu\Phi_\nu - D_\nu\Phi_\mu)^a$. This Lagrangian looks different from the Lagrangian of Eqn. (2.1), but the difference is only by a total derivative. By eliminating Φ_μ^a we will again get back the second-order Lagrangian of Eqn. (1.8).

The structure of the constraints will be different, however. The primary constraints following from this Lagrangian are

$$\begin{aligned}\Pi_0^a & \approx 0, \quad \Pi_{0i}^a \approx 0, \quad \tilde{\Pi}_0^a \approx 0, \\ \tilde{\omega}_i^a & = \tilde{\Pi}_i^a - \frac{m}{2}\epsilon_{ijk}B^{ajk} \approx 0, \\ \Lambda_{ij}^a & = \Pi_{ij}^a + m\epsilon_{oijk}A^{ak} \approx 0,\end{aligned}\quad (4.2)$$

where Π_μ^a , $\Pi_{\mu\nu}^a$, $\tilde{\Pi}_\mu^a$ are the momentum conjugates of $A^{a\mu}$, $B^{a\mu\nu}$, $\Phi^{a\mu}$ respectively. The total Hamiltonian is then

$$\begin{aligned}\mathcal{H}_T = & \frac{1}{4}F_{ij}^a F^{aij} - \frac{1}{2}\Pi_i^a \Pi^{ai} - \frac{m^2}{2}\Phi_i^a \Phi^{ai} \\ & - A_0^a \tilde{\Lambda}^a - \Phi_0^a \tilde{\omega}^a - B^{a0i} \tilde{\Lambda}_i^a,\end{aligned}\quad (4.3)$$

where $\tilde{\Lambda}^a, \tilde{\omega}^a$ and $\tilde{\Lambda}_i^a$ are the secondary constraints and are the same as the constraints denoted by the same symbols in Eqn. (2.4-2.6), each of which have non-vanishing Poisson bracket with at least one of the remaining constraints. We define the linear combinations

$$\Lambda^a = \tilde{\Lambda}^a - \frac{g}{2}f^{abc}\Lambda_{ij}^b B^{cij} - gf^{abc}\tilde{\Pi}_0^b \Phi^{c0} - gf^{abc}\tilde{\omega}_i^b \Phi^{ci}, \quad (4.4)$$

$$\bar{\Lambda}_i^a = \tilde{\Lambda}_i^a - (D^j \Lambda_{ij})^a, \quad (4.5)$$

The constraints Π_0^a , Π_{0i}^a , Λ^a , and $\bar{\Lambda}_i^a$ have vanishing (at least weakly) Poisson brackets with all constraints. The Poisson bracket of Λ^a with the canonical Hamiltonian vanishes weakly but that of $\bar{\Lambda}_i^a$ as in the previous section gives a tertiary constraint

$$\psi_i^a = \{\bar{\Lambda}_i^a, \mathcal{H}\} = gf^{abc}\Pi^{bj}\Lambda_{ij}^c - m\epsilon_{ijk}gf^{abc}\Pi^{bj}\Phi^{ck}, \quad (4.6)$$

which has non-vanishing Poisson bracket with $\bar{\Lambda}_i^a$. The Poisson brackets among the constraints are same as in Eqs. (2.9) and (2.10). Since here also we see that BFT embedding will lead only to a non-local theory, we adopt an alternate approach to modify the constraints.

As in the previous section, first we modify $\bar{\Lambda}_i^a$ to Λ_i^a such that $\{\Lambda_i^a, \mathcal{H}\} \approx 0$. As before, this modification of $\bar{\Lambda}_i^a$ changes its Poisson brackets of with all other constraints. So we further modify Λ^a , $\bar{\Lambda}_i^a$, ω^a and $\tilde{\omega}_i^a$ such that the modified constraints Λ^a , Λ_i^a are in involution with all constraints. Unlike in the earlier section here we modify the constraints by introducing canonically conjugate pairs (C_i^a, P^{bj}) , as in the BFT formalism. Thus we get the following modified first class constraints

$$\tilde{\omega}_i^a = \tilde{\Pi}_i^a - \frac{m}{2}\epsilon_{ijk}(B^{ajk} - C^{ajk}), \quad (4.7)$$

$$\Lambda^a = \tilde{\Lambda}^a - \frac{g}{2}f^{abc}\Lambda_{ij}^b B^{cij} - gf^{abc}\tilde{\Pi}_0^b \Phi^{c0} - gf^{abc}\tilde{\omega}_i^b \Phi^{ci} - gf^{abc}\chi_i^b C^{ci} \quad (4.8)$$

$$\Lambda_i^a = \tilde{\Lambda}_i^a - (D^j \Lambda_{ij})^a - \chi_i^a, \quad (4.9)$$

$$\omega^a = m^2 \Phi_0^a + \frac{m}{2}\epsilon_{ijk}D^{abi}(B^{jk} - C^{jk})^b. \quad (4.10)$$

where $C_{ij}^a = (D_i C_j - D_j C_i)^a$ and $\chi_i^a = P_i^a + \frac{m}{2}\epsilon_{ijk}\Phi^{ajk}$. At this stage we have 8 first class and 11 second class constraints.

Here we note that the combination which is left invariant by the first class constraints of Eqn. (4.9) is

$$B_{ij}^a - (D_i C_j - D_j C_i)^a \quad (4.11)$$

with the vector gauge transformations given by

$$\begin{aligned} \delta(B_{ij}^a) &= (D_i \lambda_j - D_j \lambda_i)^a, \\ \delta(C_i^a) &= \lambda_i^a. \end{aligned} \quad (4.12)$$

Obviously, the combination of Eqn. (4.11) has a further invariance, under

$$\delta(C_i^a) = (D_i\theta)^a, \quad \delta(B_{ij}^a) = gf^{abc}F_{ij}^b\theta^c. \quad (4.13)$$

Because of this invariance in Eqn. (4.13), the gauge transformation generated by Λ_i^a are not mutually independent. Since these reducible transformations of Eqs. (4.12) and (4.13) are gauge symmetries of the theory, the constraints that generate these transformations must be first class constraints. So we first enlarge the phase space by introducing another pair of conjugate variables C_0^a and P^{a0} and demand that (i) P^{a0} is a primary constraint and (ii) the total Hamiltonian of the modified theory should contain a term $-C_0^a\Theta^a$ so that Θ^a is a secondary constraint. The form of Θ^a is such that it generates the transformation (4.13) and it is linearly dependent on Λ_i^a . This fixes the form Θ^a to be

$$\Theta^a = \frac{m}{2}\epsilon_{ijk}(D^i\Phi^{jk})^a - \frac{1}{2}gf^{abc}F^{bij}\Lambda_{ij}^c - (D^i\chi_i)^a \equiv -(D^i\Lambda_i)^a. \quad (4.14)$$

which makes the generator of the transformation of Eqn. (4.12) reducible. Θ^a has vanishing Poisson brackets with all other constraints and $\{\Theta^a, \mathcal{H}\}$ gives a tertiary constraint,

$$\begin{aligned} \{\Theta^a, \mathcal{H}\} = \sigma^a = & mgf^{abc}\epsilon_{ijk}(D^i\Pi^j)^b\Phi^{ck} - gf^{abc}\Pi^{bi}\omega_i^c - gf^{abc}(D^i\Pi^j)^d\Lambda_{ij}^c \\ & - mgf^{dbc}\epsilon_{ijk}D^{adi}(\Pi^{bj}\Phi^{ck}) - m\frac{g}{2}f^{abc}\epsilon_{ijk}F^{bij}\Pi^{ck}. \end{aligned} \quad (4.15)$$

The Poisson brackets of σ^a with Λ^a and Λ_i^a vanish weakly and that with Θ^a strongly. There are no further constraints as σ^a has non-zero Poisson bracket with $\tilde{\omega}_i^a$ which makes it second-class. Thus we have obtained all the constraints of the modified theory.

Thus now the expanded theory has the first class constraints

$$\Pi_0^a, \Pi_{0i}^a, P_0^a, \Lambda^a, \Lambda_i^a, \Theta^a, \quad (4.16)$$

and second class constraints

$$\tilde{\omega}_i^a, \omega_i^a, \Lambda_{ij}^a, \omega^a, \tilde{\Pi}_0^a, \sigma^a. \quad (4.17)$$

The canonical Hamiltonian remains

$$\mathcal{H}_c = \frac{1}{4}F_{ij}^a F^{aij} - \frac{1}{2}\Pi_i^a \Pi^{ai} - \frac{m^2}{2}\Phi_i^a \Phi^{ai}. \quad (4.18)$$

The 9 linearly independent first class constraints along with 12 second class ones will leave 6 phase space degrees of freedom, and thus the model with the above constraints and \mathcal{H} describes a massive spin-one theory.

The Lagrangian from which this constraints and \mathcal{H} follow is

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{m^2}{2}\Phi_\mu^a \Phi^{a\mu} + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}\Phi^{a\mu\nu} B^{a\lambda\sigma} - \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}C^{a\mu\nu}\Phi^{a\lambda\sigma} \\ & + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}A^{a\mu}(D^\nu B^{\lambda\sigma})^a + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}A^{a\mu}\partial^\nu B^{a\lambda\sigma}, \end{aligned} \quad (4.19)$$

where $C_{\mu\nu}^a = (D_\mu C_\nu - D_\nu C_\mu)^a$. By eliminating Φ_μ^a using its equation of motion, we get a second-order Lagrangian, which up to a total derivative is

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{12}H_{\mu\nu\lambda}^a H^{a\mu\nu\lambda} + \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}B^{a\mu\nu}F^{a\lambda\sigma}, \quad (4.20)$$

where $H_{\mu\nu\lambda}^a = \partial_\mu B_{\nu\lambda}^a + gf^{abc}A_\mu^b B_{\nu\lambda}^c + gf^{abc}C_\mu^b F_{\nu\lambda}^c + \text{cyclic terms}$.

V. ROLE OF THE AUXILIARY FIELD C_μ

In this section we analyze the role of C_μ field in the dynamical 2-form theory of Eqn. (1.8). Naïvely, because of its gauge transformation property, C_μ seems to be a Stückelberg field in Eqn. (1.8) for the naïve model of Eqn. (1.4), compensating for the vector gauge symmetry of Eqn. (1.6). Usually the Stückelberg field is a dynamical field introduced to compensate for the non-invariance of the mass term under local gauge transformations. On the other hand, here the (topological) mass term in Eqn. (1.4) is invariant under the vector gauge transformation whereas the kinetic term is not. The invariance of kinetic term under Eqn. (1.6) is restored by the compensating transformation of the C_μ field. Also here the C_μ field is non-dynamical, as no kinetic term appears for it in the Lagrangian. The question that arises naturally is therefore — what is the nature of the field C_μ ?

One way of understanding the role of C_μ field is to ask if the dynamical 2-form theory Eqn. (1.8) is an embedded version of the naïve non-Abelian model of Eqn. (1.4) obtained by converting second class constraints $\bar{\Lambda}_i^a$ to first class constraints Λ_i^a as in Eqn. (4.9), which

generate the vector symmetry of Eqn. (1.6) in the extended model of Eqn. (1.8). The partition function of an embedded gauge theory is known to reduce to that of the original model with the choice of the second class constraints (T_α) of the original model as gauge fixing conditions for the first class constraints (\bar{T}_α) of the embedded model. Equivalently one can also choose the newly introduced BFT variables (here these are C_0^a, P_0^a, C_i^a and P_i^a) as the gauge fixing conditions [11,12].

Consider the phase space partition function of the dynamical 2-form theory

$$Z = \int \mathcal{D}\eta \delta(\chi^\alpha) \delta(F^\alpha) \delta(G^\beta) \Delta_{FP} \det\{\chi^\alpha, \chi^\beta\} \exp \int d^4x (P\partial^0 Q - H), \quad (5.1)$$

where the measure $\mathcal{D}\eta$ run over all phase space variables, χ^α are the second class constraints, F^α are the first class constraints, G^β are the corresponding gauge fixing conditions and Δ_{FP} is the Faddeev-Popov determinant of the embedded model. Here P and Q stand for the generic momenta and fields. Let us suppose that the dynamical 2-form theory is the embedded version of the naïve non-Abelian model. Then if we choose the second class constraints of the naïve model (either $\bar{\Lambda}_i^b$ or ψ_i^b of Eqs. (2.8), (2.11) respectively) as gauge fixing conditions corresponding to the first class constraints Λ_i^a of Eqn. (4.9), the partition function (5.1) must reduce to that of naïve non-Abelian model. For the other first class constraints of dynamical 2-form theory

$$F^\alpha = (\Pi_0^a, \Pi_{0i}^a, P_0^a, \Lambda^a), \quad (5.2)$$

we choose the gauge fixing conditions as

$$G^\alpha = (A^{b0}, B^{b0l}, C^{b0}, \partial^i A_i^b) \quad (5.3)$$

respectively. With the above choice of gauge fixing conditions (either with choice $\bar{\Lambda}_j^b$ or ψ^{bj} as the gauge conditions for Λ_i^a), it is easy to see that the Faddeev-Popov determinant vanishes. Thus the partition function does not reduce to that of naïve non-Abelian model.

Instead of choosing the original second class constraints as gauge fixing conditions, equivalently one could choose the newly introduced BFT variables as the gauge fixing conditions.

Here as we have seen, P_0^a by itself is a first class constraint (it is not appearing in any of the modified constraints) and we have chosen C^{b0} as its gauge fixing condition. We chose C^{bj} as the gauge condition for Λ_i^a of Eqn. (4.9) since their Poisson bracket is non-vanishing. But we cannot choose P_i^a as part of a gauge-fixing condition for any of the first class constraints of the dynamical 2-form theory. Consequently, we cannot implement the vanishing condition of all newly introduced BFT variables as gauge conditions and get back the original model (naïve non-Abelian model).

Thus we see that even when we choose the unitary gauge condition, the partition function of dynamical 2-form theory Eqn. (5.1) does not reduce to that of the naïve non-Abelian model. Hence the dynamical 2-form theory of Eqn. (1.8) and the naïve non-Abelian model of Eqn. (1.4) are not simply related by BFT embedding. In the covariant quantization scheme, this can be seen from the fact that implementing $C_\mu = 0$ as a gauge fixing condition for the vector symmetry (1.6) is not proper as the quadratic part of 2-form B will still be non-invertible.

VI. CONCLUSION

In this paper, we have studied three different non-Abelian generalizations of the topologically massive Abelian gauge theory in 3+1 dimensions, given in Eqn. (1.1). In Sec. II, using the canonical analysis of the naïve non-Abelian model of Eqn. (1.4), which is not invariant under the vector gauge transformations of Eqn. (1.6), we have shown that the BFT Hamiltonian embedding of this model will lead to a non-local theory. Then in Sec. III, starting with the naïve non-Abelian model and using an alternate approach we have shown that by a deformation of the constraints we can obtain the Freedman-Townsend model. Here we have used only the original phase space variables to modify the constraints. We have also shown the off-shell reducibility of the latter model. In Sec. IV, by a different modification of the constraints of naïve model, where apart from the original phase space variables newly introduced variables were also used, we have obtained the dynamical 2-form theory in the

extended phase space. We have also shown how reducibility of constraints appears in this model. In Sec. V, we have discussed the role played by the auxiliary field C_μ in the dynamical 2-form theory. Using the phase space path integral approach, we have shown that the dynamical 2-form theory cannot be obtained by a Hamiltonian embedding of the naïve non-Abelian model.

It is of interest to note the difference in Poisson bracket structures of the constraints of Freedman-Townsend model and the dynamical 2-form theory. As we have seen the constraint algebra is on-shell reducible in the case of former while in the case of latter it is off-shell reducible. In the case of Freedman-Townsend model, we see that the Poisson brackets of the scalar constraints Λ^a of Eqn. (3.1) with all other constraints vanish weakly. On the other hand, the Poisson brackets of Λ_i^a of Eqn. (3.2), which are the generators of the vector symmetry, with the remaining constraints vanish strongly like in an Abelian theory. This shows that that the model described by Eqn. (1.4) is not a pure non-Abelian theory with respect to 2-form potential, unlike the Yang-Mills gauge field. We see the same feature in the case of dynamical 2-form theory also. Here the Poisson brackets of the generators of $SU(N)$ symmetry, as given in Eqn. (4.8), with other constraints vanish weakly, while the Poisson brackets of the first class constraint of Eqn. (4.9) are strongly zero like that of an Abelian theory. Thus we see here, in the Hamiltonian formulation that the model described by Eqn. (1.8) is not a pure non-Abelian theory with respect to the 2-form potential. This is expected as the vector gauge symmetry of Eqn. (1.6) is Abelian in the case of both these models, as can be seen in the Lagrangian formulation.

It will be of interest to generalize the procedure of constraint deformation applied here so that it can be applied to other models also. Since both Freedman-Townsend model and dynamical 2-form theory also have second class constraints, it should be of interest to elevate them also to first class ones either by BFT procedure or by further deformation of constraints and study the corresponding gauge theories.

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APPENDIX A: GAUGE UNFIXING FOR THE NON-ABELIAN TWO-FORM

Apart from the generalized canonical scheme developed by Batalin, Fradkin and collaborators, there is another method to convert a system with second class constraints to gauge theory. In this method known as gauge unfixing [13,14], one modifies the second class constraint and Hamiltonian using the original phase space variables alone unlike that in the case of BFT embedding. In this scheme, among the second class constraints T_α , ($\alpha = 1, \dots, 2n$), half of them ($\bar{T}_a = T_\alpha, \alpha = 1, \dots, n$) are taken to be the constraints and the remaining half ($T_a, \alpha = n+1, \dots, 2n$) are taken as the corresponding gauge fixing conditions. Then using the constraints \bar{T}_a , a projection operator is defined using which gauge invariant Hamiltonian and other observables are constructed.

Here we apply the gauge unfixing to naïve non-Abelian model described by Eqn. (1.4). The primary constraints following from the Lagrangian of Eqn. (1.4) are

$$\Pi_0^a \approx 0, \quad \Pi_{0i}^a \approx 0, \quad (\text{A1})$$

where Π_μ and $\Pi_{\mu\nu}$ are the momenta corresponding to A^μ and $B^{\mu\nu}$ respectively. The total Hamiltonian is

$$\begin{aligned} H_T = & \frac{1}{4}\Pi_{ij}\Pi^{ij} - \frac{1}{2}\left(\Pi_i^a - \frac{m}{2}\epsilon_{0ijk}B^{ajk}\right)\left(\Pi^{ai} - \frac{m}{2}\epsilon^{0ilm}B_{lm}^a\right) + \frac{1}{4}F_{ij}^a F^{aj} - \frac{1}{12}H_{ijk}H^{ijk} \\ & - A^{a0}\left[(D^i\Pi_i)^a - \frac{1}{2}gf^{abc}\Pi_{ij}^b B^{cij}\right] + B^{a0i}\left[(D^j\Pi_{ij})^a - \frac{m}{2}\epsilon_{0ijk}F^{ajk}\right], \end{aligned} \quad (\text{A2})$$

which can also be written as $H_T = H_c - A^{a0}\Lambda^a - B^{a0i}\Lambda_i^a$, where the secondary constraints are

$$\Lambda^a = (D^i\Pi_i)^a - \frac{1}{2}gf^{abc}\Pi_{ij}^b B^{cij}, \quad (\text{A3})$$

$$\Lambda_i^a = -(D^j\Pi_{ij})^a + \frac{m}{2}\epsilon_{0ijk}F^{ajk}. \quad (\text{A4})$$

The constraint Λ^a does not lead to any further constraint as $\{\Lambda^a, H_c\} = 0$ while the persistence of Λ_i^a gives a tertiary constraint

$$\{\Lambda_i^a, H_c\} = \frac{1}{2}gf^{abc}F^{blm}H_{ilm}^c + gf^{abc}\Pi^{bl}\Pi_{il}^c + \frac{m}{2}gf^{abc}\epsilon^{0lmn}\Pi_{il}^b B_{mn}^c \equiv \Psi_i^a. \quad (\text{A5})$$

The Poisson brackets of primary constraints with all the constraints vanish and those among the remaining constraints are

$$\begin{aligned}\{\Lambda^a, \Lambda^b\} &= -g f^{abc} \Lambda^c, \quad \{\Lambda^a, \Lambda_i^b\} = g f^{abc} \Lambda_i^c, \\ \{\Lambda^a, \Psi_i^a\} &= g f^{abc} \Psi_i^c, \quad \{\Lambda_i^a, \Lambda_j^b\} = 0, \\ \{\Lambda^{ai}, \Psi_j^b\} &= -g^2 f^{bcd} f^{aed} F_{jl}^e F^{cij} - \frac{1}{2} g^2 f^{bcd} f^{aed} F_{mn}^e F^{cmn} \delta_l^i + g^2 f^{acd} f^{bce} \Pi^{dij} \Pi_{lj}^e \equiv \mathcal{A}_l^{abi}. \quad (\text{A6})\end{aligned}$$

From the constraint algebra we see that Π_0^a , Π_{0i}^a and Λ^a are first class and Λ_i^a and Ψ_i^a are second class.

From the constraint structure it is clear that for a theory which has invariance under the vector gauge transformation of Eqn. (1.6), Λ_i^a has to be a first class constraint. Thus in applying the gauge unfixing procedure we take Λ_i^a to be the first class constraint and Ψ_j^b to be the corresponding gauge fixing condition. The projection operator used to construct the gauge invariant observables is then defined as

$$\mathcal{P} = \exp - \int d^3x \Psi^{ai} \Lambda_i^a \quad (\text{A7})$$

A particular ordering is used such that when \mathcal{P} acts on functions of phase space variables, the gauge fixing condition Ψ_i^a should be outside the Poisson bracket [13,14]. Using this we construct the gauge unfixed Hamiltonian

$$\begin{aligned}\mathcal{H}_{GU} &= \mathcal{P} H_T \\ &= H_T - \int d^3y \Psi(y)^{ai} \{\Lambda_i^a, H_T\} + \frac{1}{2} \int d^3y d^3z \Psi(y)^{ai} \Psi(z)^{bj} \{\Lambda_i^a, \{\Lambda_j^b, H_T\}\} - \dots. \quad (\text{A8})\end{aligned}$$

From the constraint algebra (A6), we see that the higher order terms in the above series vanish since $\{\mathcal{A}_l^{abi}, \Lambda_j^c\} = 0$, and thus we get

$$\mathcal{H}_{GU} = H_T - \Psi_i^a \Psi^{ai} - g f^{abc} \Psi_i^a A^{0b} \Lambda^{ci} + \frac{1}{2} \Psi_i^a \Psi^{bj} \mathcal{A}_j^{abi} \quad (\text{A9})$$

But it is straight forward to check, using the constraint algebra (A6) that $\{\Lambda_i^a, \mathcal{H}_{GU}\}$ is non-vanishing and hence \mathcal{H}_{GU} is not gauge invariant. Thus here we see that the gauge unfixing method also fails to convert the naïve non-Abelian model to a first class system.

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